NOTATION

t_v, time for establishing the motion; D, a characteristic size; v, velocity of convective motion; ν, kinematic viscosity; μ, dynamic viscosity; c_p, heat capacity; k, coefficient of thermal conductivity; α, coefficient of absorption of radiation; $\chi = k/(\rho C_p)$, coefficient of thermal diffusivity; ρ, density; $Pe = vD/\chi$, Peclet's number; $Re = vD/\nu$, Reynolds number; $Pr = \mu C_p/k$, Prandtl's number; and z, height of the column of liquid.

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FLOW OF MONOTONIC RAREFIED GAS ALONG THE CLOSED PART OF THE CONTOUR OF A SLOT CHANNEL

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The asymptotic behavior of flows of strongly rarefied gas in a slot channel is constructed, and the region of applicability of the results obtained is determined for different cases of flows.

The asymptotic behavior of flows of the Heel-Shaw type constructed in [1] for a planar slot channel is unsuitable for describing he motion of rarefied gas near closed and open parts of the cylindrical surface S bounding the slot channel along the contour.

To analyze flows within the boundary layer formed near the open part of the contour of a slot channel, a dimensionless orthogonal coordinate system $\xi = x_1/H$, $\eta = x_2/H$, $\zeta = x_3/H$ is introduced, where x_1 is measured along the contour Γ, corresponding to the intersection of the surface S with the median plane S_0 of the channel, x_2 is the coordinate along the normal to the contour Γ in the plane S_0; x_3 determines the distance of this point from the plane S_0, and, H is the height of the slot channel. One can talk about the asymptotic behavior of the boundary-layer type if the effective width b of the layer in which the effect of the "special" characteristics is significant is much smaller than the scale L of the flows in the plane S_0, i.e., $b/L \ll 1$. It is evident that the curvature K of the contour Γ is of the order of L^{-1} (or less). Thus, $bK \ll 1$, while $\lambda K = O(Kn_L)$. For this reason, in order to obtain an idea of the characteristic features of flows in a boundary layer, only the first term of the expansion introduced in [1] with respect to the small parameter Kn_L need be retained, neglecting the curvature K, and therefore, the curvature of the coordinate lines η = const, i.e., the flow may be regarded as local, assuming that outside the boundary layer ($\eta \to \infty$) the flow along the ξ axis is uniform. In other words, the flow of a rarefied gas along the ξ axis is studied in the region $-\infty < \xi < \infty$, $0 < \eta < \infty$, $-0.5 < \zeta < 0.5$.

As will be shown below, flows of rarefied gas in a slot channel have the property that the long-range action, which consists of the fact that for $Kn_H \gg 1$ the effect of the boundary S extends to distances much greater than the height of the channel ($b \gg H$), while in the continuum limit $Kn_H \to 0$, $b = O(H)$. This makes it possible to restrict the analysis to states corresponding to values $\alpha = \sqrt{\pi}H/3\lambda \to 0$, when the behavior of the gas is described quite well by any model equation, including the simplest linearized Bathanger, Gross, Crook (BGC model) model [2]

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$$\varepsilon u\frac{\partial f}{\partial \xi}+v\frac{\partial f}{\partial \eta}+w\frac{\partial f}{\partial \zeta}=\alpha\left\{f_{00}\left[1+\nu+\left(u^2+v^2+w^2-\frac{3}{2}\right)\tau+2\langle u\rangle u\right]-f\right\}. \tag{1}$$

Here, for convenience, the ξ axis has been specially distinguished.

For simplicity it is assumed that the molecules reflected by the walls in the coordinate system fixed in the plates forming the slot channel have a Maxwellian distribution, corresponding to the local temperature of the wall $T_w(\xi, \eta)$, i.e., the boundary condition

$$f(\bar r,\ \zeta=\pm 0.5,\ w\lessgtr 0)=n_w(\bar r)\pi^{-3/2}h_w^{3/2}(\bar r)\exp[-h_w(\bar r)v^2] \tag{2}$$

holds.

The unknown distribution function is sought in the form of the perturbed absolutely Maxwellian distribution

$$f=f_{00}\left[1+\nu_w+\left(u^2+v^2+w^2-\frac{3}{2}\right)\tau_w+\Phi\right]. \tag{3}$$

If absorption and emission of molecules do not occur at the walls of the channel, while the temperature of the walls depends linearly on ξ, then, as usual, in the analysis of one-dimensional flows in a planar slot, symmetric with respect to the median plane, the variation of the temperature and number density of molecules in the transverse direction can be neglected, i.e., it may be assumed that $\partial\tau/\partial\zeta = 0$, $\partial\nu/\partial\zeta = 0$. Then, by virtue of (1), the perturbation Φ will satisfy the equation

$$\varepsilon u\frac{\partial \Phi}{\partial \xi}+v\frac{\partial \Phi}{\partial \eta}+w\frac{\partial \Phi}{\partial \zeta}+\alpha\Phi+u\left[k_\nu+\left(u^2+v^2+w^2-\frac{3}{2}\right)k_\tau-2\alpha\langle u\rangle\right]=0. \tag{4}$$

Integrating the last equation from the bottom wall to the top wall and from the surface $S(\eta = 0)$ to an arbitrary point $\bar r_1(\xi, \eta, \zeta)$ along the characteristics $u^{-1}d\xi = \varepsilon v^{-1}d\eta = \varepsilon w^{-1}d\zeta$, assuming that the mass velocity $\langle u\rangle$ entering into (4) is a known function of the coordinates, taking into account the uniform boundary conditions at the walls $[\Phi = \Phi_1(\xi, \eta, \zeta = \pm 0.5, w \lessgtr 0) = 0$, $\Phi = \Phi_2(\xi, \eta = 0, \zeta, v > 0) = 0]$ we obtain

$$\Phi_1(w\gtrless 0)=-\int_{\mp 0,5}^{\xi} u\left[k_\nu+\left(u^2+v^2+w^2-\frac{3}{2}\right)k_\tau-2\alpha\langle u\rangle(\eta',\zeta')\right]\exp\left[-\frac{\alpha}{w}(\zeta-\zeta')\right]\frac{d\zeta'}{w}, \tag{5}$$

$$\Phi_2(v>0)=-\int_0^{\eta} u\left[k_\nu+\left(u^2+v^2+w^2-\frac{3}{2}\right)k_\tau-2\alpha\langle u\rangle(\eta',\zeta')\right]\exp\left[-\frac{\alpha}{v}(\eta-\eta')\right]\frac{d\eta'}{v}.$$

The moments corresponding to the hydrodynamic flow parameters are calculated using the formula

$$\begin{aligned} M(\psi)&=\pi^{-3/2}\iiint_{-\infty}^{\infty}\psi\Phi\exp(-V^2)\,d\bar V=\pi^{-3/2}\int_{-\infty}^{\infty}\exp(-u^2)\,du\ \times\\ &\times\left\{\int_0^{\infty}\exp(-w^2)\,dw\int_{w\eta/(0,5+\zeta)}^{\infty}\psi\Phi_2\exp(-v^2)\,dv+\int_{-\infty}^{\infty}\exp(-w^2)\,dw\ \times\right.\\ &\times\int_{-w\eta/(0,5-\zeta)}^{\infty}\psi\Phi_2\exp(-v^2)\,dv+\int_0^{\infty}\exp(-v^2)\,dv\left[\int_{-\infty}^{-v(0,5-\zeta)/\eta}\psi\Phi_1\exp(-w^2)\,dw+\right.\\ &\left.\left.+\int_{-v(0,5+\zeta)/\eta}^{\infty}\psi\Phi_1\exp(-w^2)\,dw\right]+\int_{-\infty}^{0}\exp(-v^2)\,dv\int_{-\infty}^{\infty}\psi\Phi_1\exp(-w^2)\,dw\right\}. \end{aligned} \tag{6}$$

For $\psi = u$, the last relation transforms into an integral equation for the mass velocity $\langle u\rangle(\eta, \zeta)$ near the closed part of the frame of the slot channel, containing in the integrand the unknown function $\langle u\rangle(\eta', \zeta') = \langle u\rangle[\eta - v/w(\zeta - \zeta'), \zeta']$ multiplied by the coefficient α. It will be shown below that wh en $\alpha \ll 1$ within the "boundary" (kinetic) layer the velocity averaged over the height of the slot channel increases almost two-fold. For this reason, in order to estimate the effect of this layer on the flow in the slot channel it is desirable to determine its effective width, introducing the displacement width b_* by analogy with the displacement width used in the Prandtl boundary-layer theory for a viscous liquid with $Re \gg 1$. A strict definition of this quantity will be given below, and it will be known that when

$\alpha \ll 1$ $b_* \gg H$. It is under these conditions that the significance of the kinetic layer under study is manifested and one can talk about the effect of long-range action in a flow of rarefied gas. But in the limit $\alpha \to 0$ the role of the term $2\alpha\langle u\rangle(\eta', \zeta')$ in the integral equation, obtained from (6) using (5), becomes insignificant. Indeed, the linearity of the problem under study implies that

$$\langle u \rangle = k_\nu U_\nu(\eta, \zeta) + k_\tau U_\tau(\eta, \zeta), \quad U_{\nu,\tau} = U_{\nu,\tau}(\infty, \zeta) + \delta U_{\nu,\tau}(\eta, \zeta). \tag{7}$$

Here $U_{\nu,\tau}(\infty, \zeta)$ corresponds to the flow outside the boundary layer under study, i.e., in the limit $\eta \to \infty$ $\delta U_{\nu,\tau}(\eta, \zeta) \to 0$. Substitution of the expressions (5) and (7) into (6) gives two integral equations for determining the dimensionless coefficients $\delta U_\nu(\eta, \zeta)$ and $\delta U_\tau(\eta, \zeta)$, whose kernels have a logarithmic singularity at $\zeta' = \zeta$ ($\eta' = \eta$), like in the analysis of Poiseuille's flows between infinite plates. Therefore, following [3], in the functions $U_{\nu,\tau}(\eta', \zeta')$ the arguments η', ζ' can be replaced by η, ζ and these functions can be removed from the integrand, which will make it possible to obtain the corresponding approximate solution in an explicit form. For flows between infinite plates this approach gives results which are close to the exact solution, in any case for $\alpha \leqslant 1$ [3]. Here, however, we are interested in states corresponding to $\alpha \ll 1$.

The functions $U_{\nu,\tau}(\eta, \zeta, \alpha)$ averaged over the height or the slot channel can also be represented by sums of the type

$$Q_{\nu,\tau}(\eta, \alpha) = \int_{-0.5}^{0.5} U_{\nu,\tau}(\eta, \zeta, \alpha)\, d\zeta = Q_{\nu,\tau}(\infty, \alpha) + \delta Q_{\nu,\tau}(\eta, \alpha). \tag{8}$$

The indicated substitutions and changes in the order of integration in the multiple integrals lead to the following relations:

$$\delta Q_{\nu,\tau}(\eta, \alpha) = \frac{\delta Q^*_{\nu,\tau}(\eta, \alpha) - 2\alpha Q_{\nu,\tau}(\infty, \alpha)\,\delta Q^*_\nu(\eta, \alpha)}{1 + 2\alpha\delta Q^*_\nu(\eta, \alpha)}; \tag{9}$$

$$\delta Q^*_\nu(\eta, \alpha) = \frac{1}{\pi\alpha} \int_0^\infty \exp(-v^2)\, dv \int_0^{v/\eta} \exp(-w^2) \left[\exp\left(-\frac{\alpha\eta}{v}\right) - \frac{w}{v}\eta \times \right.$$
$$\left. \times \exp\left(-\frac{\alpha\eta}{v}\right) - \frac{w}{\alpha}\exp\left(-\frac{\alpha\eta}{v}\right) + \frac{w}{\alpha}\exp\left(-\frac{\alpha}{w}\right)\right] dw = \sum_{j=1}^{4} \delta Q_j; \tag{10}$$

$$\delta Q^*_\tau(\eta, \alpha) = \frac{2}{\pi^{3/2}\alpha} \int_{-\infty}^{\infty} u^2 \left(u^2 + v^2 + w^2 - \frac{3}{2}\right) \exp(-u^2)\, du \int_0^\infty \exp(-v^2)\, dv \times$$
$$\times \int_0^{v/\eta} \left[\exp\left(-\frac{\alpha\eta}{v}\right) - \frac{w}{v}\eta\exp\left(-\frac{\alpha\eta}{v}\right) - \frac{w}{\alpha}\exp\left(-\frac{\alpha\eta}{v}\right) + \right.$$
$$\left. + \frac{w}{\alpha}\exp\left(-\frac{\alpha}{w}\right)\right] \exp(-w^2)\, dw = \sum_{i,\, j=1}^{4} \delta Q_{ij}, \tag{11}$$

where δQ_j corresponds to the j-th term in the brackets of the integrand in (10), while δQ_{ij} corresponds to the product of the i-th term in the brackets of the first integral in (11) by the j-th term in the brackets of the latter:

$$\delta Q_1 = \frac{1}{2\sqrt{\pi}\alpha} \int_0^\infty \operatorname{erf}\left(\frac{v}{\eta}\right) \exp\left(-v^2 - \frac{\alpha\eta}{v}\right) dv,$$
$$\delta Q_2 = -\frac{\eta}{2\pi\alpha}\left[J_{-1}(\alpha\eta) - J_{-1}(\alpha\sqrt{1+\eta^2})\right], \tag{12}$$
$$\delta Q_3 = -\frac{1}{2\pi\alpha^2}\left[J_0(\alpha\eta) - \frac{\eta J_0(\alpha\sqrt{1+\eta^2})}{\sqrt{1+\eta^2}}\right],$$
$$\delta Q_4 = \frac{1}{2\sqrt{\pi}\alpha^2} \int_0^\infty [1 - \operatorname{erf}(w\eta)]\, w \exp\left(-w^2 - \frac{\alpha}{w}\right) dw.$$

The values of the integrals $J_m(x)=\int_0^\infty v^m \exp\left(-v^2-\frac{x}{v}\right)dv$ are tabulated in [4].

At the boundary $\eta = 0$

$$\delta Q_1=\frac{1}{4\alpha},\quad \delta Q_2=0,\quad \delta Q_3=\frac{1}{4\sqrt{\pi}\alpha^2},\quad \delta Q_4=\frac{J_1(\alpha)}{2\sqrt{\pi}\alpha}$$

and in the limit $\alpha \to 0$, as shown in [5]:

$$J_1(\alpha)=\frac{1}{2}-\frac{\sqrt{\pi}}{2}\alpha-\frac{\alpha^2}{2}\left(\ln\alpha-\frac{1}{2}\right)+\ldots \tag{13}$$

Thus for large Kn_H numbers (Kn_H ($\alpha \to 0$) on the surface S of the frame ($\eta = 0$)

$$\delta Q_v^*(0,\alpha)\simeq-\frac{1}{4\sqrt{\pi}}\left(\ln\alpha-\frac{1}{2}\right)=\frac{1}{2}Q_v'(\infty,\alpha). \tag{14}$$

Here Q_v (∞, α) corresponds to the asymptotic representation of the solution of the problem of a Poiseuille flow between infinite plates for the BGC model [2] in the limit $\alpha \to 0$.

It is easy to show that $\delta Q_{1j} + \delta Q_{4j} = 0$ ($j = 1, 2, 3, 4$), while

$$\delta Q_{21}=\frac{1}{2\sqrt{\pi}\,\alpha}\int_0^\infty \operatorname{erf}\left(\frac{v}{\eta}\right)\exp\left(-v^2-\frac{\alpha\eta}{v}\right)v^2dv,$$

$$\delta Q_{31}=\frac{1}{4\sqrt{\pi}\,\alpha}\int_0^\infty \operatorname{erf}\left(\frac{v}{\eta}\right)\exp\left(-v^2-\frac{\alpha\eta}{v}\right)dv-\frac{\eta J_1(\alpha\sqrt{\eta^2+1})}{2\pi\alpha(\eta^2+1)},$$

$$\delta Q_{22}=-\frac{\eta}{2\pi\alpha}J_1(\alpha\eta)+\frac{\eta^3 J_1(\alpha\sqrt{\eta^2+1})}{2\pi\alpha(\eta^2+1)},$$

$$\delta Q_{32}=-\frac{\eta}{2\pi\alpha}\left[J_{-1}(\alpha\eta)-J_{-1}(\alpha\sqrt{\eta^2+1})-\frac{J_1(\alpha\sqrt{\eta^2+1})}{\eta^2+1}\right], \tag{15}$$

$$\delta Q_{23}=-\frac{1}{2\pi\alpha^2}\left[J_2(\alpha\eta)-\frac{\eta^3 J_2(\alpha\sqrt{\eta^2+1})}{(\eta^2+1)^{3/2}}\right],$$

$$\delta Q_{33}=-\frac{1}{2\pi\alpha^2}\left[J_0(\alpha\eta)-\frac{\eta J_0(\alpha\sqrt{\eta^2+1})}{\sqrt{\eta^2+1}}-\frac{\eta J_2(\alpha\sqrt{\eta^2+1})}{(\eta+1)^{3/2}}\right],$$

$$\delta Q_{24}=\frac{1}{2\pi\alpha^2}\left\{\frac{\sqrt{\pi}}{2}\int_0^\infty[1-\operatorname{erf}(w\eta)]\exp\left(-w^2-\frac{\alpha}{w}\right)wdw+\frac{\eta J_2(\alpha\sqrt{\eta^2+1})}{(\eta^2+1)^{3/2}}\right\},$$

$$\delta Q_{34}=\frac{1}{2\sqrt{\pi}\,\alpha_2}\int_0^\infty[1-\operatorname{erf}(w\eta)]\exp\left(-w^2-\frac{\alpha}{w}\right)w^3dw.$$

At the boundary $S(\eta = 0)$.

$$\delta Q_\tau^*(0,\alpha)=\sum_{i,j=1}^{4}\delta Q_{ij}=\frac{1}{4\alpha}+\frac{1}{8\alpha^2\sqrt{\pi}}[2J_1(\alpha)+4J_3(\alpha)-3].$$

In the limit $\alpha \to 0$ the asymptotic formulas (13) and $J_3(\alpha) = 1/2 - \sqrt{\pi}/4\ \alpha + \alpha^2/4 + \ldots$ can be used, which makes it possible to rewrite the latter relation in the form

$$\delta Q_\tau^*(0,\alpha)\simeq-\frac{1}{8\sqrt{\pi}}\left(\ln\alpha-\frac{3}{2}\right)=\frac{1}{2}Q_\tau'(\infty,\alpha). \tag{16}$$

Here $Q_\tau'(\infty, \alpha)$ corresponds to the asymptotic representation of the solution of the problem of a flow between infinite plates under the action of a temperature gradient with $k_v = 0$ [6]. It is evident from the relations (9), (14), and (16) that in the limit $\alpha \to 0$ in the boundary layer the average macroscopic velocity increases approximately twofold. For large Kn_H numbers, as is evident from the table, $Q_v \approx Q_v^*$. The latter circumstance is an additional justification for replacing the integral equation by a finite expression for the macroscopic velocity, i.e., removing this velocity from the integrand, since the contribution corresponding to this term to the function $Q_v(\eta, \alpha)$ is very small.

TABLE 1. Values of the Functions $Q_\nu/Q_{\nu\infty}$ and $Q_\nu^*/Q_{\nu\infty}$ as a Function of the Kn_H Number

η	$Kn_H = 30$		$Kn_H = 60$		$Kn_H = 100$	
	$Q_\nu/Q_{\nu\infty}$	$Q_\nu^*/Q_{\nu\infty}$	$Q_\nu/Q_{\nu\infty}$	$Q_\nu^*/Q_{\nu\infty}$	$Q_\nu/Q_{\nu\infty}$	$Q_\nu^*/Q_{\nu\infty}$
0	0,4880	0,5000	0,4925	0,5000	0,4953	0,5000
0,1	0,5390	0,5509	0,5375	0,5450	0,5365	0,5411
0,5	0,6489	0,6597	0,6340	0,6410	0,6250	0,6293
1	0,7140	0,7237	0,6919	0,9983	0,6783	0,6824
4	0,8459	0,8521	0,8131	0,8176	0,7915	0,7946
8	0,9020	0,9062	0,8685	0,8719	0,8452	0,8476
14	0,9389	0,9416	0,9081	0,9105	0,8849	0,8868
30	0,9742	0,9755	0,9514	0,9527	0,9315	0,9327
60	0,9916	0,9920	0,9788	0,9637	0,9637	0,9644

The indicated property substantially simplifies the calculation of the displacement width of the boundary layer b_*, which by analogy with the displacement thickness of a Prandtl boundary layer of a viscous liquid, we define by the relation

$$b_*\left[Q_\nu(\infty,\ \alpha)k_\nu + Q_\tau(\infty,\ \alpha)k_\tau\right] = H\left[\Delta_\nu(\alpha)k_\nu + \Delta_\tau k_\tau\right],$$
$$\Delta_{\nu,\tau}(\alpha) = \int_0^\infty \delta Q_{\nu,\tau}(\eta,\ \alpha)\,d\eta,\quad \Delta_\nu = \sum_{j=1}^{4}\Delta_j,\quad \Delta_\tau = \sum_{i=2}^{3}\sum_{j=1}^{4}\Delta_{ij}, \tag{17}$$
$$\Delta_j = \int_0^\infty \delta Q_j(\eta,\ \alpha)\,d\eta,\quad \Delta_{ij} = \int_0^\infty \delta Q_{ij}(\eta,\ \alpha)\,d\eta.$$

Here the functions $\delta Q_{\nu,\tau}$ are replaced by $\delta Q^*_{\nu,\tau}$. By virtue of (12) and (15).

$$\Delta_1 = \frac{1}{2\pi\alpha^2}\left[\frac{\sqrt{\pi}}{2} - J_0(\alpha)\right],\quad \Delta_2 = \frac{1}{2\pi\alpha^3}\left[J_1(0) - \alpha J_0(\alpha) - J_1(\alpha)\right],$$
$$\Delta_3 = \frac{1}{2\pi\alpha^3}\left[J_1(\alpha) - J_1(0)\right],\quad \Delta_4 = \frac{J_0(\alpha)}{2\pi\alpha^2},$$
$$\Delta_{21} = \frac{1}{2\pi\alpha^2}\left[\frac{\sqrt{\pi}}{2} - J_0(\alpha)\right],\quad \Delta_{31} = \frac{1}{2\pi\alpha^2}\left[\frac{\sqrt{\pi}}{4} - J_2(\alpha)\right],$$
$$\Delta_{22} + \Delta_{32} = \frac{1}{2\pi\alpha^2}\left\{J_3(\alpha) + J_1(\alpha) - 1 + \alpha\left[J_2(\alpha) + J_0(\alpha)\right]\right\}, \tag{18}$$
$$\Delta_{23} + \Delta_{33} = \frac{1}{2\pi\alpha^2}\left[J_3(\alpha) + J_1(\alpha) - J_3(0) - J_1(0)\right],$$
$$\Delta_{24} = \frac{J_0(\alpha)}{2\pi\alpha^2},\quad \Delta_{34} = \frac{J_2(\alpha)}{2\pi\alpha^2}.$$

Using the asymptotic formula (5) for the integrals $J_m(\alpha)$ in the limit $\alpha \to 0$, we obtain

$$\Delta_1 \approx -\frac{\ln\alpha}{2\pi\alpha},\quad \Delta_2 \approx \frac{\ln\alpha + 0.5}{4\pi\alpha},\quad \Delta_3 \approx \frac{-1}{4\sqrt{\pi}\,\alpha} - \frac{\ln\alpha - 0.5}{4\pi\alpha},$$
$$\Delta_4 \approx \frac{1}{4\sqrt{\pi}\,\alpha^2} + \frac{\ln\alpha}{2\pi\alpha},\quad \Delta_{21} \approx -\frac{\ln\alpha}{2\pi\alpha},\quad \Delta_{31} \approx \frac{1}{4\pi\alpha}, \tag{19}$$
$$\Delta_{22} + \Delta_{32} \approx -\frac{\ln\alpha}{4\pi\alpha},\quad \Delta_{23} + \Delta_{33} \approx \frac{-1}{2\pi\alpha^2}\left[\frac{3\sqrt{\pi}}{4} + \frac{\alpha}{2}(\ln\alpha - 1)\right],$$
$$\Delta_{24} + \Delta_{34} \approx \frac{1}{2\pi\alpha^2}\left[\frac{3\sqrt{\pi}}{4} + \alpha\left(\ln\alpha - \frac{1}{2}\right)\right].$$

Thus for large Kn_H ($\alpha \ll 1$) numbers $\Delta_\nu \approx \Delta_\tau \approx (4\pi\alpha)^{-1}$, and therefore the following formula for the displacement width follows from the relation (17)

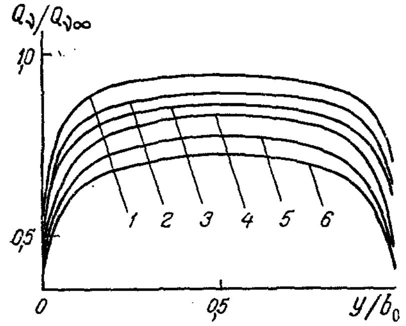

Fig. 1. Variation of the velocity of the gas, averaged over the height of the slot channel, with a relative channel width of $\eta_0' = b_0/H = 60$ and Knudsen numbers $Kn_H = 30$ (1); 60 (2); 100 (3); for $\eta_0' = 20$ and $Kn_H = 30$ (4); 60 (5) and 100 (6).

$$b_* = -\Delta_\nu[(1-\theta)Q_{\nu\infty} + \theta Q_{\tau\infty}]^{-1}H, \quad \theta = \frac{k_\tau}{k_\text{п}},$$

$$k_\text{п} = k_\nu + k_\tau = \frac{1}{p}\frac{dp}{d\xi}. \tag{20}$$

But in the limit $\alpha \to 0$

$$Q_{\nu\infty} \approx \frac{\ln\alpha - 0.5}{2\sqrt{\pi}}, \quad Q_{\tau\infty} \approx \frac{\ln\alpha - 1.5}{4\sqrt{\pi}}. \tag{21}$$

Therefore

$$\frac{b_*}{H} \approx \left\{\sqrt{\pi}\,\alpha\left[1 + \frac{\theta}{2} - (2-\theta)\ln\alpha\right]\right\}^{-1}, \tag{22}$$

$$\frac{b_*}{\lambda} \approx 3\left\{\pi\alpha\left[1 + \frac{\theta}{2} - (2-\theta)\ln\alpha\right]\right\}^{-1}. \tag{23}$$

In the limit $\alpha \to 0$ $b_*/H \to \infty$, as $(\alpha \ln \alpha)^{-1}$ (long-range action effect). In the limit $\theta \to -\infty$, $\alpha = \text{const}$ $b_*/\lambda \to 0$, $b_*/H \to 0$.

Once the displacement width b_* is determined, different problems concerning flows of a rarefied gas in a slot channel with $0 < Kn_H < \infty$ and $Kn_L \ll 1$ can be solved. All results obtained above are valid under the condition that the displacement width b_*, determined by the formulas (22) and (23), is much smaller than the linear scale L, characterizing the unperturbed, or external relative to the boundary layer under study, flow in the symmetry plane of the gap. For example, for a slot channel formed by two semiinfinite plates, whose edges are connected by a flat plane ($y = \eta = 0$), while the gradients of the number density of the molecules and temperature are everywhere parallel to the indicated edges, the scale $L = \infty$ and the proposed asymptotic behavior is valid for arbitrarily large Kn_H numbers. If, on the other hand, the corresponding edges of the plates are connected by perpendicualr plates with a cylindrical surface, then the scale L must be taken as the minimum value of the radius of curvature of the line of intersection of the plates with the cylindrical surface and the region of applicability of the results obtained above is limited above by the number $Kn_H \ll Kn_{H*}$, where Kn_{H*} is determined by the value $b_* = L$, calculated from the formulas (22) or (23).

For the flow of a gas in a planar channel of finite width the corresponding width b_0 is the scale ($L = b_0$). In the latter case, it is easy to pass through the above-indicated limit, and exceed the boundary-layer approximation. The geometric simplicity of such a channel makes it possible to take into account the effect of all walls at any point of the transverse cross section. For this, the boundary conditions presented above, must be supplemented by conditions on the lateral wall: $y = b_0 = L$ ($\eta = \eta_0 = b_0/H$), $\Phi = \Phi_3(v < 0) = 0$. Then the elements of the phase space, corresponding to characteristics which begin on the lateral wall $\eta = \eta_0$, correspond to the perturbation $\Phi_3(u, v, w, \xi, \eta, \zeta) = \Phi_2(u, v_1, w, \eta_1, \zeta)$, where $v_1 = -v$, $\eta_1 = \eta_0 - \eta$, while the moment of any hydrodynamic parameter ψ can be written in the form

$$M(\psi) = \frac{1}{\sqrt{\pi^3}}\int_{-\infty}^{\infty}\exp(-u^2)\,du\left\{\int_0^{\infty}\exp(-w^2)\,dw \times\right.$$

$$\times\left[\int_{w\eta/(0,5+\zeta)}^{\infty}\psi\Phi_2(v,\ \eta)\exp(-v^2)\,dv+\int_{w\eta_1/(0,5+\zeta)}^{\infty}\psi\Phi_2(v_1,\ \eta_1)\exp(-v_1^2)dv_1\right]+$$
$$\int_{-\infty}^{0}\exp(-w^2)\,dw\left[\int_{-w\eta/(0,5-\zeta)}^{\infty}\psi\Phi_2(v,\ \eta)\exp(-v^2)\,dv+\right.$$
$$\left.+\int_{-w\eta_1/(0,5-\zeta)}^{\infty}\psi\Phi_2(v_1,\ \eta_1)\exp(-v_1^2)\,dv_1\right]+\int_0^{\infty}\exp(-v^2)\,dv\times \tag{24}$$
$$\times\left[\int_{-\infty}^{-v(0,5-\zeta)/\eta}\psi\Phi_1(v,\ \eta)\exp(-w^2)\,dw+\int_{v(0,5+\zeta)/\eta}^{\infty}\psi\Phi_1(v,\ \eta)\exp(-w^2)\,dw\right]+$$
$$\left.+\int_0^{\infty}\exp(-v_1^2)\,dv_1\left[\int_{-\infty}^{-v_1(0,5-\zeta)/\eta_1}\psi\Phi_1(v_1,\ \eta_1)\exp(-w^2)\,dw+\int_{v_1(0,5+\zeta)/\eta_1}^{\infty}\psi\Phi_1\exp(-w^2)\,dw\right]\right\}.$$

From a comparison of this expression with the expression (6) or the boundary-layer approximation we obtain formulas for the local velocities and the velocities averaged over the height of the channel:

$$\begin{gathered}U_{\nu,\tau}(\eta,\ \zeta,\ \alpha)=U_{\nu,\tau\infty}(\zeta,\ \alpha)+\delta U^0_{\nu,\tau}(\eta,\ \zeta,\ \alpha),\\ Q_{\nu,\tau}(\eta,\ \alpha)=Q_{\nu,\tau\infty}(\alpha)+\delta Q^0_{\nu,\tau}(\eta,\ \alpha),\\ \delta U^0_{\nu,\tau}(\eta,\ \zeta,\ \alpha)=\delta U_{\nu,\tau}(\eta,\ \zeta,\ \alpha)+\delta U_{\nu,\tau}(\eta_1,\ \zeta,\ \alpha),\\ \delta Q^0_{\nu,\tau}(\eta,\ \alpha)=\delta Q_{\nu,\tau}(\eta,\ \alpha)+\delta Q_{\nu,\tau}(\eta_1,\ \alpha).\end{gathered} \tag{25}$$

Thus the functions $\delta U_{\nu,\tau}$, $\delta Q_{\nu,\tau}$, determined above, can be used not only in the boundary-layer approximation, but also for calculating the velocity field in a planar channel, whose width b_0 is comparable and even less than the displacement width b*. The results of this calculation are presented in Fig. 1, whence it is evident that for large Kn_H numbers decreasing the relative width of the channel η_0 effectively reduces the value of the gas velocity averaged over the section. As the relative width of the channel is increased or Bond's number Bo_H is decreased, the profile of the velocities averaged over the height of the channel is appreciably straightened, and a steep boundary layer forms at the side walls.

NOTATION

H, height of the slot channel; L, linear scale in the median plane, $Kn_H = \lambda/H$; $Kn_L = \lambda/L$; λ, average mean-free path length of the molecules; $\alpha = \sqrt{\pi}\,H/3\lambda$; f, distribution function; f_{00}, absolute Maxwellian distribution; $f_{00} = n^*/2\pi RT^* \exp(-V^2/2RT^*)$; $\bar{V}$, instantaneous velocity vector of the molecules $\bar{V} = V_j\bar{e}_j$, $u = \sqrt{h}\,V_1$, $v = \sqrt{h}\,V_2$, $w = \sqrt{h}\,V_3$; $h = (2RT^*)^{-1}$; x_1, x_2, x_3, Cartesian coordinates: x_1, along the contour, x_2, distance from the contour, and x_3, distance from the median plane of the slot channel; $\xi = x_1/H$; $\eta = x_2/H$; $\xi = x_3/H$; T*, characteristic temperature; n*, characteristic number density of the molecules; T, temperature; n, number density of molecules; $\nu = (n - n^*)/n^*$; $\tau = (T - T^*)/T^*$; $k_\nu = \partial\nu/\partial\xi$; $k_\tau = \partial\tau/\partial\xi$; $\langle u\rangle$, macroscopic gas velocity oriented along the ξ axis; Φ, perturbation of the distribution function; $U_\nu(\eta,\ \zeta)$, $U_\tau(\eta,\ \zeta)$, $\delta U_{\nu,\tau}(\eta,\ \zeta)$, velocity coefficients defined in (7); $Q_{\nu,\tau}(\eta,\ \zeta)$ and $\delta Q_{\nu,\tau}(\eta,\ \zeta)$, are the coefficients defined in (8); δQ_i and δQ_{ij}, coefficients defined in (10) and (11); $Q_{\nu\infty} = Q_{\nu,\tau}(\infty,\ \alpha)$; b*, displacement width of the boundary layer; $\Theta = k_\tau/k_{II}$; $k_{II} = (1/p)(\partial p/\partial\xi) = k_\nu + k_\tau$.

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